Efficient run-time dispatching in generic programming with minimal code bloat

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Generic programming with C++ templates results in efficient but inflexible code: efficient, because the exact types of inputs to generic functions are known at compile time; inflexible because they must be known at compile time. We show how to achieve run-time polymorphism without compromising performance by instantiating the generic algorithm with a comprehensive set of possible parameter types, and choosing the appropriate instantiation at run time. Applying this approach naively can result in excessive template bloat: a large number of template instantiations, many of which are identical at the assembly level. We show practical examples of this approach quickly approaching the limits of the compiler. Consequently, we combine this method of run-time polymorphism for generic programming, with a strategy for reducing the number of necessary template instantiations. We report on using our approach in GIL, Adobe’s open source Generic Image Library. We observed a notable reduction, up to 70% at times, in executable sizes of our test programs. This was the case even with compilers that perform aggressive template hoisting at the compiler level, due to significantly smaller dispatching code. The framework draws from both the generic and generative programming paradigms, using static metaprogramming to fine tune the compilation of a generic library. Our test bed, GIL, is deployed in a real world industrial setting, where code size is often an important factor.

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1. Introduction

Generic programming, pioneered by Musser and Stepanov [21], and introduced to C++ with the STL [26], aims at expressing algorithms at an abstract level, such that the algorithms apply to as broad a class of data types as possible. A key idea of generic programming is that abstraction should incur no performance degradation: once a generic algorithm is specialized for some concrete data types, its performance should not differ from a similar algorithm written directly for those data types. This principle is often referred to as zero abstraction penalty. The paradigm of generic programming has been successfully applied in C++, evidenced, e.g., by the STL, the Boost Graph Library (BGL) [23], and many other generic libraries [3,6,12,22,24,25]. One factor contributing to this success is the compilation model of C++ templates, where specialized code is generated for every different instance of a template. We refer to this compilation model as the instantiation model.

We note that the instantiation model is not the only mechanism for compiling generic definitions. For example, in Java [14] and Eiffel [11] a generic definition is compiled to a single piece of byte or native code, used by all instantiations
of the generic definition. C# [10,19] and the ECMA .NET framework delay the instantiation of generics until run time. Such alternative compilation models address the code bloat issue, but may be less efficient or may require run-time compilation. They are not discussed in this paper.

With the instantiation model, zero abstraction penalty is an attainable goal: later phases of the compilation process make no distinction between code generated from a template instantiation, and non-template code written directly by the programmer. Thus, function calls can be resolved statically, which enables inlining and other optimizations for generic code. The instantiation model, however, has other less desirable characteristics, which we focus on in this paper.

In many applications, the exact types of objects to be passed to generic algorithms are not known at compile time. In C++ all template instantiations, and code generation that they trigger, occur at compile time—dynamic dispatching to templated functions is not (directly) supported. For efficiency, however, it may be crucial to use an algorithm instantiated for particular concrete types.

In this paper, we describe how to instantiate a generic algorithm with all possible types it may be called with, and generate code that dispatches at run time to the right instantiation. This approach combines the flexibility of dynamic dispatching, and performance typical for the instantiation model: the dispatching occurs only once per call to a generic algorithm, and has thus a negligible cost, whereas the individual instantiations of the algorithms are compiled and fully optimized, knowing their concrete input types. This solution, however, leads easily to an excessive number of template instantiations, a problem known as code bloat or template bloat. In the instantiation model, the combined size of the instantiations grows with the number of instantiations: there is typically no code sharing between instantiations of the same templates with different types, regardless of how similar the generated code is. One exception is Microsoft’s Visual Studio 8 compiler, which can reuse the body of assembly-level identical functions. In Section 6 we demonstrate that our method results in noticeable code size reduction, even in the presence of this optimization.

This paper reports on experiences of using the generic programming paradigm in the development of the Generic Image Library (GIL) [6] in the Adobe Source Libraries [2]. GIL supports several image formats, each represented internally with a distinct type. The static type of an image manipulated by an application using GIL is often not known: the type assigned to an image may, e.g., depend on the format it was stored on the disk. Thus, the case described above manifests in GIL: an application using GIL must instantiate the relevant generic functions for all possible image types, and arrange that the correct instantiations are selected based on the arguments’ dynamic types, when calling these functions. Following this strategy blindly may lead to unmanageable code bloat, as we mentioned above. In particular, the set of instantiations increases exponentially with the number of image type parameters that can be varied independently in an algorithm. Our experience shows that the number of template instantiations is an important design criterion in developing generic libraries.

We describe the techniques and the design we use in GIL, to ensure that specialized code for all performance critical program parts is generated, but still keep the number of template instantiations low. Our solution is based on the realization that, even though a generic function is instantiated with different type arguments, the generated code is in some cases identical. We describe mechanisms that allow different instantiations to be replaced with a single common instantiation. The basic idea is to decompose a complex type into a set of orthogonal parameter dimensions (with image types, these include color space, channel depth, and constness) and identify the dimensions that matter for instantiations of a given generic algorithm. Dimensions irrelevant for a given operation can be cast to a single “base” parameter value. Note that while this technique is presented as a solution to dealing with code bloat originating from the “dynamic dispatching” we use in GIL, the technique can also be useful in generic libraries that do not need to resort to dynamic dispatching.

We note that the approach of telescoping languages [20] shares some of our goals, namely avoiding run-time dispatching (and other even more costly things, such as executing an interpreter) in performance critical algorithms. Telescoping languages have mainly been used to speed up scripting languages, such as that of MATLAB. They use a host of techniques to attain the above goals, primarily compiling and optimizing specialized instances of library routines during a library preprocessing step, and augmenting the script compiler with the knowledge of these specialized routines. Also, we can view our techniques as performing program specialization via offline partial evaluation [13]—some of the inputs (or types of input) are fixed for each concrete instance of an algorithm, and the optimizing C++ compiler is utilized to generate fast residual programs.

In general, a developer of software libraries and the technologies supporting library development are faced with many competing challenges, originating from the vastly different potential uses for the libraries. Considering GIL, for example, an application such as Adobe Photoshop requires a library flexible enough to handle the variation of image representations at run time, but also places strict constraints on performance. Small memory footprint becomes essential when using GIL as part of a software running on a small device, such as a cellular phone or a PDA. Basic software engineering principles ask for easy extensibility. The design and techniques presented in this paper help in building generic libraries that can combine efficiency, flexibility, extensibility, and compactness.

The structure of the paper is as follows. Section 2 describes typical approaches to fighting code bloat. Section 3 gives a brief introduction to GIL, and the code bloat problems therein. Section 4 explains the mechanism we use to tackle code bloat, and Section 5 describes how to apply the mechanism with dynamic dispatching to generic algorithms. We report experimental results in Section 6, and conclude in Section 7. Since GIL is work in progress, we have created a snapshot of the library matching the discussion in the paper, available at http://download.macromedia.com/pub/opensource/gil/scp08.zip. At present, the type reduction mechanism described in Section 4 has not yet been ported to the current release version of GIL (2.0).
2. Background

One common strategy to reduce code bloat in the instantiation model is template hoisting (see e.g. [7]). In this approach, a class template is split into a non-generic base class, and a generic derived class. Every member function that does not depend on any of the template parameters is moved, hoisted, into the base class; also non-member functions can be defined to operate directly on references or pointers to objects of the base-class type. As a result, the amount of code that must be generated for each different instantiation of the derived class decreases. For example, red-black trees are used in the implementation of associative containers map, multimap, set, and multiset in the C++ Standard Library [16]. Because the tree balancing code does not need to depend on the types of the elements contained in these containers, a high-quality implementation is expected to hoist this functionality to non-generic functions. The GNU Standard C++ Library v3 does exactly this: the tree balancing functions operate on pointers to a non-generic base class of the tree’s node type.

In the case of associative containers, the tree node type is split into a generic and non-generic part. It is, in principle, possible to split a template class into several layers of base classes, such that each layer reduces the number of template parameters. Each layer then potentially has less type variability than its subclasses, and thus two different instantiations of the most derived class may coalesce to a common instantiation of a base class. Such designs seem to be rare.

Template hoisting within a class hierarchy is a useful technique, but it allows only a single way of splitting a data type into sub-parts. Different generic algorithms are generally concerned with different aspects of a data-type. Splitting a data type in a certain way may suit one algorithm, but not help to reduce instantiations of other algorithms. In the framework discussed in this paper, the library developer, possibly also the client of a library, can define a partition of data-types, where a particular algorithm needs to be instantiated only with one representative for each equivalence class in the partition.

We define the partition such that differences between types that do not affect the operation of an algorithm are ignored. One common example is pointers—for some algorithms the pointed-to type is important, whereas for others viewing the pointer type as void* is sufficient. Another example is differences due to constness (consider STL’s iterator and const_iterator concept). The generated code for invoking a non-modifying algorithm (one which accepts immutable iterators) with mutable iterators, will be identical to the code generated for an invocation with immutable iterators. Further, some algorithms operate on raw bit representation of data, others depend on the type of data. For example, assignment between a pair of pixels is the same regardless of whether they are CMYK or RGBA pixels, whereas the type of pixels matters to an algorithm that sets their color.

C++’s template system provides a programmable sub-language for encoding compile-time computations, the uses of which are known as template metaprogramming (see e.g. [27],[9, Section 10]). This form of generative programming proved to be crucial in our solution: the process of pruning unnecessary instantiations is orchestrated with template metaprograms. In particular, for our metaprogramming needs, we use the Boost Metaprogramming Library (MPL) [1,15] extensively. In the presentation, we assume some familiarity with the basic principles of template metaprogramming in C++.

3. Generic image library

The Generic Image Library (GIL) is Adobe’s open source image processing library [6], and part of the C++ Boost [5] collection of peer-reviewed C++ libraries. GIL addresses a fundamental problem in image processing projects—operations applied to images (such as copying, comparing, or applying a convolution) are logically the same for all image types, but in practice image representations in memory vary significantly. Consequently, often multiple variations of the same algorithm are necessary. GIL is used as the framework for a new feature planned for inclusion in the Adobe Photoshop CS3. GIL is also being adopted in several other imaging projects inside Adobe.

Images are 2D (or more generally, n-dimensional) arrays of pixels. Each pixel encodes the color at a particular point in the image. The color is typically represented as the values of a set of color channels, whose interpretation is defined by a color space. For example, the color red can be represented as 100% red, 0% green, and 0% blue using the RGB color space; its approximation in the CMYK color space is 0% cyan, 96% magenta, 90% yellow, and 0% black. Typically all pixels in an image are represented with the same color space.

GIL must support significant variation within image representations. Besides color space, images vary in the ordering of the channels in memory (RGB vs. BGR), and in the number of bits (depth) and representation (8 bit vs. 32 bit, unsigned char vs. float) of each color channel. Image data may be in interleaved form (RGBRGBRGB...) or in planar form where each color plane is separate in memory (RRR..., GGG... BBB...); some algorithms are more efficient in planar form, whereas others perform better in interleaved form. In some image representations, each row (or the color planes) may be aligned, in which case a gap of unused bytes may be present at the end of each row. There are representations where pixels are not consecutive in memory, such as a sub-sampled view of another image that only considers every other pixel. The image may represent a rectangular sub-image in another image, or an upside-down view of another image. The pixels of the image may require an arbitrary transformation (for example an 8-bit RGB view of 16-bit CMYK data). The image data may not be in memory at all (a virtual image, or an image inside a JPEG file). The image may be synthetic, defined by an arbitrary function (the Mandelbrot set), and so forth.

Note that GIL makes a distinction between images and image views. Images are containers that own their pixels, views do not. Images can return their associated views and GIL algorithms operate on views. For the purpose of this paper, these differences are not significant, and we use the terms image and image view (or just view), interchangeably.
The exact image representation is irrelevant to many image processing algorithms. To compare two images we need to loop over the pixels and compare them pairwise. To copy one image into another, we need to copy every pixel pairwise. To compute the histogram of an image, we need to accumulate the histogram data over all pixels. To exploit these commonalities, GIL follows the generic programming approach, exemplified by the STL, and defines abstract representations of images as concepts. In the terminology of generic programming, a concept is the formalization of an abstraction as a set of requirements on a type (or types) \[4,17\]. A type that implements the requirements of a concept is said to model the concept. Algorithms written in terms of image concepts work for images in any representation that model the necessary concepts. By this means, GIL avoids multiple definitions for the same algorithms that merely accommodate for inessential variation in the image representations.

GIL supports a multitude of image representations, for each of which a distinct typedef is provided. Examples of these types are rgb8_view_t (8-bit mutable interleaved RGB image), bgr16c_view_t (16-bit immutable interleaved BGR image), cmyk32_planar_view_t (32-bit mutable planar CMYK image), and lab8c_step_planar_view_t (8-bit immutable LAB planar image in which the pixels are not consecutive in memory). The actual types associated with these typedefs are somewhat involved and omitted here.

GIL represents color spaces with distinct types. The naming of these types is as expected: rgb_t stands for the RGB color space, cmyk_t for the CMYK color space, and so forth. Channels can be represented in different permutations of the same set of color values. For each set of color values, GIL identifies a single color space as the primary color space—its permutations are derived color spaces. For example, rgb_t is a primary color space and bgr_t is its derived color space.

GIL defines two images to be compatible if they have the same set and type of channels. That also implies their color spaces must have the same primary color space. Compatible images may vary in other ways: planar vs. interleaved organization, mutability, etc. For example, an 8-bit RGB planar image is compatible with an 8-bit BGR interleaved image. Compatible images may be copied from one another and compared for equality.

3.1. GIL algorithms

We demonstrate the operation of GIL with a simple algorithm, copy_pixels(), that copies one image view to another. Below is one way to implement it. Note that GIL image views do not own the pixels and do not propagate their constness to the pixels, which explains why we take the destination as a const reference. Mutability is incorporated into the image view type.

```cpp
template<typename View1, typename View2>
void copy_pixels(const View1& src, const View2& dst) {
    std::copy(src.begin(), src.end(), dst.begin());
}
```

A requirement of copy_pixels is that the two image view types be compatible, and have the same dimensions, and that the destination be mutable. An attempt to instantiate copy_pixels with incompatible image types results in a compile-time error. Each GIL image type supports the `begin()` and `end()` member functions as defined in the STL’s Container concept, and thus the body of the algorithm can leverage the `copy()` algorithm from the C++ standard library. If we expand out the std::copy() function, copy_pixels becomes:

```cpp
template<typename View1, typename View2>
void copy_pixels(const View1& src, const View2& dst) {
    typename View1::iterator src_it = src.begin();
    typename View2::iterator dst_it = dst.begin();
    while (src_it != dst.end()) { *dst_it++ = *src_it++; }
}
```

Each image type is required to have an associated iterator type, that implements iteration over the image’s pixels. Furthermore, each pixel type must support assignment. Note that the source and target images can be of different (albeit compatible) types, and thus the assignment may include a (lossless) conversion from one pixel type to another. These elementary operations are implemented differently by different image types. A built-in pointer type can serve as the iterator type of a simple interleaved image, whereas in a planar RGB image the iterator may be a bundle of three pointers, one to each color plane. The iterator increment operator ++ for interleaved images may resolve to a pointer increment, for step images to advancing a pointer by a given number of bytes, and for a planar RGB image to incrementing three pointers. The dereferencing operator * for simple interleaved images returns a reference type; for planar RGB images it returns a planar reference proxy object containing three references to the three channels. For a complex image type, such as one representing an RGB view over CMYK data, the dereferencing operator may perform color conversion.

Due to the instantiation model, the calls to the implementations of the elementary image operations in GIL algorithms can be resolved statically, and usually inlined, resulting in an efficient algorithm, specialized for the particular image types used. GIL algorithms are targeted to match the performance of code hand-written for a particular image type. Any difference in performance from that of hand-written code is usually due to abstraction penalty, for example, the compiler failing to
inline a forwarding function, or failing to put small objects of user-defined types in registers. Modern compilers exhibit zero abstraction penalty with GIL algorithms in many common uses of the library.

3.2. Dynamic dispatching in GIL

Often, the exact image type with which an algorithm is to be called is unknown at compile time. For this purpose, GIL provides the variant template, which implements a discriminated union type, capable of storing values of any type from a given list of types. GIL variant accepts exactly one template argument that specifies this list. A suitable argument is, for example, the vector template in the MPL, a compile-time data structure whose elements are types.

Populating a variant with image types, and instantiating another template in GIL, any_image_view, with the variant, yields a GIL image type that can hold any of the image types in the variant. Note the difference to polymorphism via inheritance and dynamic dispatching: in polymorphism via virtual member functions, the set of virtual member functions, and thus the set of algorithms, is fixed but the set of data types implementing those algorithms is extensible; with variant types, the set of data types is fixed, but there is no limit to the number of algorithms that can be defined for those data types. The following code illustrates the use of the any_image_view type:

```cpp
typedef variant<mpl::vector<rgb8_view_t, bgr16c_view_t, cmyk32_planar_view_t, lab8_step_planar_view_t>> my_views_t;
any_image_view<my_views_t> v1, v2;
jpeg_read_view(file_name1, v1);
jpeg_read_view(file_name2, v2);
...;
copy_pixels(v1, v2);
```

Compiling the call to copy_pixels involves examining the dynamic types of v1 and v2, and dispatching to the instantiation of copy_pixels generated for those types. Indeed, GIL overloads its generic algorithms for any_image_view types to do exactly this. Consequently, all run-time dispatching occurs at the level of entering algorithms, rather than at the inner loops of the algorithms; any_image_view containers are practically as efficient as if the exact image type were known at compile time. Obviously, the precondition to dispatching to a specific instantiation, is that the instantiation has been generated. Unless we are careful, this may lead to significant template bloat, as illustrated in the next section.

3.3. Template bloat originating from GIL's dynamic dispatching

To ease the definition of lists of types for the any_image_view template, GIL implements type generators. One of these generators is cross_vector_image_view_types, which generates all image types that are combinations of given sets of color spaces and channels, and the interleaved/planar and step/no step policies, as the following example demonstrates:

```cpp
typedef mpl::vector<rgb_t, bgr_t, lab_t, cmyk_t>::type ColorSpaceV;
typedef mpl::vector<bits8, bits16, bits32>::type ChannelV;
typedef any_image_view<
    cross_vector_image_view_types<
        ColorSpaceV, ChannelV, kInterleavedAndPlanar, kNonStepAndStep
    >::type
    > any_view_t;
any_view_t v1, v2;
v1 = rgb8_planar_view_t(..);
v2 = bgr8_view_t(..);
copy_pixels(v1, v2);
```

This code defines any_view_t to be one of $4 \times 3 \times 2 \times 2 = 48$ possible image types. It can have any of the four listed color spaces, any of the three listed channel depths, it can be interleaved or planar, and its pixels can be adjacent or non-adjacent in memory. The code generates $48 \times 48 = 2304$ instantiations. Without any special handling, the code bloat will be out of control.

In practice, the majority of these combinations are between incompatible images; calling copy_pixels with incompatible images triggers an exception. Nevertheless, such exhaustive code generation is wasteful, since many of the cases generate essentially identical code. For example, copying two 8-bit interleaved RGB images or two 8-bit interleaved LAB images (with the same channel types) results in the same assembly code—the interpretation of the channels is irrelevant for the copy operation. The following section describes how we can use metaprograms to avoid generating such identical instantiations.
4. Reducing the number of instantiations

Our strategy for reducing the number of instantiations is based on decomposing a complex type into a set of orthogonal parameter dimensions (such as color space, channel depth, constness), and identifying which dimensions are important for a given operation. Dimensions irrelevant for a given operation can be cast to a single "base" parameter value. For example, for the purpose of copying, all LAB and RGB images could be treated as RGB images. As mentioned in Section 2, for each algorithm we define a partition among the data types, select the equivalence class representatives, and only generate an instance of the algorithm for these representatives. We call this process type reduction.

Type reduction is implemented with metafunctions, which map a given data type and a particular algorithm to the class representative of that data type for the given algorithm. By default, this reduction is identity:

```cpp
template<typename Op, typename T>
struct reduce { typedef T type; };
```

By providing template specializations of the reduce template for specific types, the library author can define the partition of types for each algorithm. We return to this point later. Note that the algorithm is represented with the type Op here; we implement GIL algorithms internally as function objects, instead of free-standing function templates. One advantage of function objects is that we can represent an algorithm with a template parameter.

We need a generic way of invoking an algorithm which will apply the reduce metafunction to perform type reduction on its arguments, prior to entering the body of the algorithm. For this purpose, we define the apply_operation function:

```cpp
template<typename Arg, typename Op>
inline typename Op::result_type apply_operation(const Arg& arg, Op op) {
    typedef typename reduce<Op, Arg>::type base_t;
    return op(reinterpret_cast<const base_t&>(arg));
}
```

This function provides the glue between our technique and the algorithm. We have overloads for the one and two argument cases, and overloads for variant types. Note that reinterpret_cast is not portable. To cast between two arbitrary types GIL uses instead the more portable expression static_cast<T*>(static_cast<void*>(arg)). We omit this detail for readability.

The apply_operation function serves two purposes: it applies reduction to the arguments, and invokes the associated function. As the example above illustrates, for templated types the second step amounts to a simple function call. In Section 5, we will see that for variants this second step also resolves the static types of the objects stored in the variants, by going through a switch statement.

Consider an example algorithm, invert_pixels. It inverts each channel of each pixel in an image. Fig. 1 shows a possible implementation (which ignores performance and focuses on simplicity) that can be invoked via apply_operation.

With the definitions this far, nothing has changed from the perspective of the library's client. The invert_pixels() function merely forwards its parameter to apply_operation(), which again forwards to invert_pixels_op(). Both apply_operation() and invert_pixels() are inlined, and the end result is the same as if the algorithm implementation were written directly in the body of invert_pixels(). With this arrangement, however, we can control instantiations with defining specializations for the
reduce metafunction. For example, the following statement will cause 8-bit LAB images to be reduced to 8-bit RGB images when calling invert_pixels:

```cpp
template<>
struct reduce<invert_pixels_op, lab8_view_t> {
    typedef rgb8_view_t type;
};
```

This approach extends to algorithms taking more than one argument—all arguments can be represented jointly as a tuple. The reduce metafunction for binary algorithms can have specializations for `std::pair` of any two image types the algorithm can be called with, as we show in Section 4.1. The space of all pairs of input types, however, can be very large. In particular, using variant types as arguments to binary algorithms (see Section 5), generates a large number of such pair types, which can take a toll on compile time. Fortunately, for many binary algorithms, it is possible to apply unary reduction independently on each of the input arguments first, and only consider pairs of the argument types after reduction—potentially a much smaller set of pairs. We call such preliminary unary reduction pre-reduction. The `apply_operation` for algorithms taking two image arguments is as follows:

```cpp
template <typename Arg1, typename Arg2, typename Op>
inline typename Op::result_type
apply_operation(const Arg1& arg1, const Arg2& arg2, Op op) {
    // unary pre-reduction
    typedef typename reduce<Op, Arg1>::type base1_t;
    typedef typename reduce<Op, Arg2>::type base2_t;
    // binary reduction
    typedef std::pair<const base1_t*, const base2_t*> pair_t;
    typedef typename reduce<Op, pair_t>::type base_pair_t;
    std::pair<const void*, const void*> p(&arg1, &arg2);
    return op(reinterpret_cast<const base_pair_t&>(p));
}
```

As a concrete example of a binary algorithm that can be invoked via `apply_operation`, the `copy_pixels()` function can be defined as follows:

```cpp
struct copy_pixels_op {
    typedef void result_type;
    template <typename View1, typename View2>
    void operator() (const std::pair<const View1*, const View2*>& p) const {
        typename View1::iterator src_it = p.first->begin();
        typename View2::iterator dst_it = p.second->begin();
        while (dst_it != dst.end()) *dst_it++ = *src_it++;
    }
};
template <typename View1, typename View2>
inline void copy_pixels(const View1& src, const View2& dst) {
    apply_operation(src, dst, copy_pixels_op());
}
```

We note that the type reduction mechanism relies on an unsafe cast operation, which relies on programmers assumptions not checked by the compiler, or the run time system. The library author defining the `reduce` metafunction must thus know the implementation details of the class representative, and types that are being mapped to it. A client of the library defining new image types can specialize the `reduce` template, to specify a partition within those types, without the need to understand the implementations of existing image types in the library.

### 4.1. Defining reduction metafunctions

The reduce metafunction can be implemented by whatever means is most suitable, most straightforwardly by enumerating all cases separately. Often a more concise definition is possible, and we can identify “helper” metafunctions that can be reused in type reduction for many algorithms. To demonstrate, we describe our implementation for the type reduction of the `copy_pixels` algorithm. Even though we use MPL in GIL extensively, following the definitions requires no knowledge of MPL; here we use a traditional static metaprogramming style of C++, where branching is expressed with partial specializations.
The `copy_pixels` algorithm operates on two images—we thus apply the two-phase reduction strategy discussed in Section 4, first pre-reducing each image independently, then applying pair-wise reduction.

To define the type reductions for GIL image types, `reduce` must be specialized for them:

```cpp
template<typename Op, typename L>
struct reduce<Op, image_view<L>> :
  public reduce_view_basic<Op, image_view<L>> :::value {};

template<typename Op, typename L1, typename L2>
struct reduce<Op, std::pair<
const image_view<L1>*, const image_view<L2>*>> :
  public reduce_views_basic<Op, image_view<L1>,
image_view<L2>, view_is_basic<image_view<L1>>::value &&
view_is_basic<image_view<L2>>::value> {};
```

Note the use of `metafunction forwarding` idiom from the MPL, where one metafunction is defined in terms of another metafunction by inheriting from it; here the two specializations of `reduce` are defined in terms of `reduce_view_basic` and `reduce_views_basic`, respectively.

The first of the above specializations will match any GIL `image_view` type, the second any pair of GIL `image_view` types. We represent the two types as a pair of constant pointers, because it makes the implementation of reduction with a variant (described in Section 5) easier. The above specializations do no more than forward to `reduce_view_basic` and `reduce_views_basic`—two metafunctions, specific to reducing GIL's image view types. The `view_is_basic` template defines a compile time predicate, that tests whether a given view type is one of GIL's built-in view types, rather than one defined by the client of the library. We can only define the reductions of view types known to the library, the ones satisfying the predicate—for all other types GIL applies identity mappings using the following default definitions for `reduce_view_basic` and `reduce_views_basic`:

```cpp
template<typename Op, typename View, bool IsBasic>
struct reduce_view_basic {
  typedef View type; }

template<>
struct reduce_view_basic<lab_t> {
  typedef rgb_t type; }

template<>
struct reduce_view_basic<hsb_t> {
  typedef rgb_t type; }

template<>
struct reduce_view_basic<cmyk_t> {
  typedef rgba_t type; }
```

We can similarly define a binary color space reduction—a metafunction that takes a pair of (compatible) color spaces and returns a pair of reduced color spaces. For brevity, we only show the interface of the metafunction:

```cpp
template<typename Cs> struct reduce_color_space {
  typedef Cs t;
}
template<>
struct reduce_color_space<lab_t> {
  typedef rgb_t t;
}
template<>
struct reduce_color_space<hsb_t> {
  typedef rgb_t t;
}
template<>
struct reduce_color_space<cmyk_t> {
  typedef rgb_t t;
}
```

The equivalence classes defined by this metafunction represent the color space pairs, where the mapping of channels from first to second color space is preserved. We can represent such mappings with a tuple of integers. For example, the mapping of `pair<rgb_t, bgr_t>` is `⟨2, 1, 0⟩`, as the first channel `r` maps from the position 0 to position 2, `g` from position 1 to 1, and `b` from 2 to 1. Mappings for `pair<bgr_t, bgr_t>` and `pair<lab_t, lab_t>` are represented with the tuple `⟨0, 1, 2⟩`. We have identified eight mappings that can represent nearly all pairs of color spaces that are used in practice. New mappings can be introduced when needed as specializations.

With the above helper metafunctions, we can define the type reduction for `copy_pixels`. First, unary pre-reduction is performed for each image view type independently to reduce the color spaces with the `reduce_color_space` metafunction, and to unify both mutable and immutable views. We use GIL's `derived_view_type` metafunction (we omit the definition for brevity) that takes a source image view type, and returns a related image view in which some of the parameters are different; here, we change color space and mutability.
template <typename V1, typename V2>
class reduce_copy_pixop_compat<V1, V2, true> {
    typedef typename V1::color_space_t Cs1;
    typedef typename V2::color_space_t Cs2;
    typedef typename reduce_color_spaces<Cs1, Cs2>::first_t RCs1;
    typedef typename reduce_color_spaces<Cs1, Cs2>::second_t RCs2;
    typedef typename derived_view_type<V1, use_default, RCs1,
        use_default, use_default, mpl::false_>::type RV1;
    typedef typename derived_view_type<V2, use_default, RCs2,
        use_default, use_default, mpl::true_>::type RV2;
    public:
        typedef std::pair<
            const RV1*, const RV2*>
            type;
};

Fig. 2. Type reduction for copy_pixels of compatible images.

template <typename V>
class reduce_view_basic<copy_pixels_op, View, true> {
    typedef typename reduce_color_space<typename View::color_space_t>::type Cs;
    public:
        typedef typename derived_view_type<
            V, use_default, Cs, use_default, use_default, mpl::true_>
            ::type type;
};

Mutability is specified by the last parameter of derived_view_type and
mpl::true_ is a type corresponding to the boolean value true. Note that
this reduction introduces a slight problem, as it would allow us to copy (incorrectly)
between some incompatible images, for example from hsb8_view_t into lab8_view_t as they both will be reduced to rgb8_view_t. Such calls should never occur, as calling copy_pixels with incompatible images violates its precondition. This pre-reduce significantly
improves compile times. Due to the above objection, however, we did not use it in our measured experiments.

The first step of binary reduction is to check whether the two images are compatible; the views_are_compatible predicate provides this information. If the images are not compatible, we reduce to error_t—a special tag denoting type mismatch error. All algorithms throw an exception when given error_t:

template <typename V1, typename V2>
struct reduce_views_basic<copy_pixels_op, V1, V2, true>
    : public reduce_copy_pixop_compat<V1, V2, views_are_compatible<V1, V2>::value &&
        view_is_mutable<V2>::value > {};

template <typename V1, typename V2, bool IsCompatible>
struct reduce_copy_pixop_compat (typedef error_t type);

Finally, if the two image views are compatible, we reduce their color spaces pairwise, using the reduce_color_spaces metafunction discussed above. Fig. 2 shows the code where the metafunction derived_view_type again generates the reduced view types, potentially changing color spaces, making the source immutable and the destination mutable, but keeping other aspects of the image view types the same.

Note that we can easily reuse the type reduction policy of copy_pixels for other algorithms for which the same policy applies:

template <typename V, bool IsBasic>
struct reduce_view_basic<resample_view_op, V, IsBasic>
    : public reduce_view_basic<copy_pixels_op, V, IsBasic> {};

template <typename V1, typename V2, bool AreBasic>
struct reduce_views_basic<resample_view_op, V1, V2, AreBasic>
    : public reduce_views_basic<copy_pixels_op, V1, V2, AreBasic> {};

5. Minimizing instantiations with variants

Type reduction is most necessary, and most effective with variant types, such as GIL’s any_image_view, as a single invocation of a generic algorithm would otherwise require instantiations to be generated for all types in the variant, or
even for all combinations of types drawn from several variants. This section describes how we apply the type reduction machinery in the case of variant types.

Variants in GIL are comprised of three elements—a type vector of possible types the variant can store (Types), a run-time value (index) to this vector, indicating the type of the object currently stored in the variant, and the memory block containing the instantiated object (bits). Invoking an algorithm, which we represent as a function object, amounts to a switch statement over the value of index, each case N of which casts bits to the Nth element of Types and passes the cast value to the function object. We capture this functionality in the apply_operation_base template, shown below. The number of cases in the switch statement is equal to the size of the Types vector. We use the preprocessor, the Boost Preprocessor Library in particular [18], to generate such functions with different numbers of case statements, and select the correct one at compile time, using template specialization.

```
template <typename Types, typename Bits, typename Op>
type_op::result_type apply_operation(const variant<Types>& arg, Op op) {
    return unary_reduce<Types, Op>::apply(arg._bits, arg._index, op);
}
```

As we discussed before, such code instantiates the algorithm with every possible type and can lead to code bloat. Instead of calling this function directly from the apply_operation function template overloaded for variants, we first subject the Types vector to reduction:

```
template <typename Types, typename Bits, typename Op>
type_op::result_type apply_operation_base(const Bits& bits, int index, Op op) {
    switch (index) {
        ... case N: return op(reinterpret_cast<const typename mpl::at_c<Types, N>::type&>(bits));
        ... }
}
```

The unary_reduce template performs type reduction, and its apply member function invokes apply_operation_base with the smaller, reduced, set of types. The definition of unary_reduce is shown in Fig. 3. The definitions of the three typedefs are omitted; they represent the following intermediate results:

- reduced_t: a type vector that holds the reduced types corresponding to each element of Types. That is, reduced_t[i] == reduce<Op, Types[i]>::type
- unique_t: a type set containing the same elements as the type vector reduced_t, but without duplicates.
- indices_t: a type set containing the indices (represented as MPL integral types, which wrap integral constants into types) mapping the reduced_t vector onto the unique_t set, i.e., reduced_t[i] == unique_t[indices_t[i]]

The dynamic_at_c function is parameterized with a type vector of MPL integral types. It takes an index to the type vector and returns the element in the type vector as a run-time value. That is, we are using a run-time index to get a run-time value out from a type vector. The definitions of dynamic_at_c function are generated with the preprocessor; the code looks similar to the code below. In reality the number of table entries must equal the size of the type vector. We use the Boost Preprocessor Library to generate function objects specialized over the size of the type vector, whose application operators generate tables.
template <typename Types1, typename Types2, typename Op>
struct binary_reduction {
    typedef typename unary_reduce<Types1, Op>::::unique_t unique1_t;
    typedef typename unary_reduce<Types2, Op>::::unique_t unique2_t;
    typedef cross_product_pairs<unique1_t, unique2_t> bin_types;
    typedef typename unary_reduce<bin_types, Op>::::unique_t unique_t;

    static inline int map_indices(int index1, int index2) {
        int r1 = unary_reduce<Types1, Op>::::map_index(index1);
        int r2 = unary_reduce<Types2, Op>::::map_index(index2);
        return unary_reduce<bin_types, Op>::::map_index(r2 * mpl::size<unique1_t>::value + r1);
    }

public:
    template <typename Bits1, typename Bits2>
    static typename Op::result_type
    apply(const Bits1& bits1, int index1, const Bits2& bits2, int index2, Op op) {
        std::pair<const void*, const void*> pr(&bits1, &bits2);
        return apply_operation_base<unique_t>(pr, map_indices(index1, index2), op);
    }
};

template <typename T1, typename T2, typename BinOp>
inline typename BinOp::result_type
apply_operation(const variant<T1>& arg1, const variant<T2>& arg2, BinOp op) {
    return binary_reduce<T1, T2, Op>::::apply(arg1_bits, arg1_index, arg2_bits, arg2_index, op);
}

Fig. 4. Binary reduction for variant types.

of appropriate sizes and perform the lookup. We dispatch to the right specialization at compile time, thereby assuring the most compact table is generated.

template <typename Ints>
static int dynamic_at_c(int index) {
    static int table[] = {
        mpl::at_c<Ints, 0>::value,
        mpl::at_c<Ints, 1>::value,
        ...,
    }
    return table[index];
}

Some algorithms, like copy_pixels, can possibly take two variant arguments. Without any type reduction, a binary variant operation is implemented using a double-dispatch: first invoke apply_operation_base with the first variant, passing it a function object, which, when invoked will in turn call apply_operation_base on the second argument, passing it the original function. If $N$ is the number of types in each input variant, this implementation will generate $N^2$ instantiations of the algorithm and $N + 1$ switch statements, having $N$ cases each. Considering the argument types together, rather than each independently, can potentially lead to more reduction.

Fig. 4 shows the definition of the overload for the binary apply_operation function template. We leave several details without discussion, but the general strategy can be observed from the code:

(i) Perform unary_reduce on each input argument to obtain the set of unique reduced types, unique1_t and unique2_t. A binary algorithm can define pre-reductions for its argument types, such as the color space reductions described in Section 4.1. Any pre-reductions at this step are beneficial, as they reduce the amount of compile-time computations preformed in the next step.

(ii) Compute bin_types, a type vector for the cross-product of the unique pre-reduced types. Its elements are all the instances std::pair<const T1*, const T2*> with T1 and T2 drawn from unique1_t and unique2_t respectively.

(iii) Perform unary reduction on bin_types, to obtain unique_t—the set of unique pairs after reducing each pair under the binary operation.
Finally, to invoke the binary operation, we use a switch statement over the unique pairs of types left over after reduction. We map the two indices to the corresponding single index over the unique set of pairs. This version is advantageous, because it instantiates far fewer than $N^2$ types and uses a single switch statement instead of two nested ones.

6. Experimental results

To assess the effectiveness of type reduction in practice, we measured the executable sizes and compilation times, with and without type reduction in programs that called GIL algorithms with objects of variant types.

6.1. Compiler settings

For our experiments we used the C++ compilers of GCC 4.0 on OS X 10.4 and Visual Studio 8 on Windows XP. For GCC we used the optimization flag –O2, and removed the symbol information from the executables with the Unix strip command prior to measuring executable size. Visual Studio 8 was set to compile in release mode, using all settings that can help reduce code size, in particular the “Minimize Size” optimization (/O1), link-time code generation (/Gl), and eliminating unreferenced data (/OPT:REF). With these settings, the compiler can in some cases, detect that two different instances of template functions generate the same code, and avoid the duplication of that code. This makes template bloat a lesser problem in the Visual Studio compiler, as type reduction possibly occurs directly in the compiler. We show, however, improvement even with the most aggressive code-size minimization settings.

6.2. Test images

For testing type reduction with unary operations, we used GIL image views that can vary in color space (Grayscale, RGB, BGR, LAB, HSB, CMYK, RGBA, ABGR, BGRA, ARGB), in channel depth (8-bit, 16-bit and 32-bit) and in whether the pixels are consecutive in memory or offset by a run-time specified step. This amounts to $10 \times 3 \times 2 = 60$ combinations of interleaved images. In addition, we include planar versions for the primary color spaces (RGB, LAB, HSB, CMYK and RGBA) which adds another $5 \times 3 \times 2 = 30$ combinations for a total of 90 image types. Note that we split the images in two sets because GIL does not allow planar versions of grayscale (they are identical to interleaved), or derived color spaces (they can be represented by the primary color spaces by rearranging the order of the pointers to the color planes in the image construction).

For binary operations, we use two smaller test sets. Test B consists of ten images: Grayscale, BGR, RGB, step RGB, planar RGB, planar step RGB, LAB, step LAB, planar LAB, planar step LAB, all of which are in 8-bit. Test C consists of twelve 8-bit images: RGB, LAB and HSB, each of which can be planar or interleaved, step or non-step. Smaller sets are used, because binary operations result in an explosion in the number of combinations to consider for type reduction. The practical upper limit for direct reduction, with today’s compilers and typical desktop computers, is about $20 \times 20$ combinations; much beyond that consumes undesirable amounts of compilation resources. Note that GIL determines how complex a given binary type reduction will be, and suppresses computing it directly when the number of combinations exceeds a limit. In such a case, the binary operation is represented via double-dispatch as two nested unary operations. This allows more complex binary functions to compile, but the type reduction may miss some possibilities for sharing instantiations.

To summarize: the test set A contains 90 image types, B contains 10 image types, and C contains 12 image types.

6.3. Test algorithms

We tested with three algorithms—invert_pixels, copy_pixels and resample_view.

The unary algorithm invert_pixels inverts each channel of each pixel in an image. Although less useful than other algorithms, invert_pixels is simple, and allows us to measure the effect of our technique, without introducing too much GIL-related code. As a channel-independent operation, invert_pixels does not depend on the color space or ordering of the channels. We tested invert_pixels with the test set A: type reduction maps the 90 image types in this set down to 30 equivalence classes.

The copy_pixels algorithm, as discussed in Sections 3 and 4, is a binary algorithm that performs a channel-wise copy between compatible images, and throws an exception when invoked with incompatible images. Applied to test images B, our reduction for copy_pixels reduces the image pair types from $10 \times 10 = 100$ down to 26 (25 plus one “incompatible image” case). Without this reduction, there are 42 compatible combinations and 58 incompatible ones. The code for the invalid combinations is likely to be shared, even without reduction. Thus our reduction transforms 43 cases into 26 cases, which is approximately a 40% reduction.

For test images C, our reduction for copy_pixels reduces the image pairs from $12 \times 12 = 144$ down to 17 (16 plus the “incompatible image” case). Without the reduction, there would be 48 valid and 96 invalid combinations. Thus our reduction transforms 49 cases into 17 cases, which is approximately a 65% reduction.

The resample_view is also a binary operation. It resamples the destination image from the source under an arbitrary geometric transformation, and interpolates the results using bicubic, bilinear, or nearest-neighbor methods. It is a little more involved than copy_pixels, and therefore less likely to be inlined. The reduction rules are the same as those of copy_pixels.
Table 1
Size, in kilobytes, of the generated executable in the five test programs compiled with (a) GCC 4.0 and (b) Visual Studio 8’s C++ compilers, without (Sn) and with (Sr) type reduction

<table>
<thead>
<tr>
<th></th>
<th>Sn</th>
<th>Sr</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>201.6</td>
<td>107.5</td>
<td>47</td>
</tr>
<tr>
<td>Test 2</td>
<td>252.8</td>
<td>75.9</td>
<td>70</td>
</tr>
<tr>
<td>Test 3</td>
<td>259.8</td>
<td>144.0</td>
<td>45</td>
</tr>
<tr>
<td>Test 4</td>
<td>318.7</td>
<td>98.8</td>
<td>69</td>
</tr>
<tr>
<td>Test 5</td>
<td>62.2</td>
<td>31.2</td>
<td>50</td>
</tr>
</tbody>
</table>

(b) Sn Sr Decrease (%)

<table>
<thead>
<tr>
<th></th>
<th>Sn</th>
<th>Sr</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>42.0</td>
<td>26.0</td>
<td>37</td>
</tr>
<tr>
<td>Test 2</td>
<td>40.0</td>
<td>25.0</td>
<td>37</td>
</tr>
<tr>
<td>Test 3</td>
<td>46.0</td>
<td>42.0</td>
<td>8</td>
</tr>
<tr>
<td>Test 4</td>
<td>33.5</td>
<td>34.0</td>
<td>-1</td>
</tr>
<tr>
<td>Test 5</td>
<td>24.0</td>
<td>16.5</td>
<td>31</td>
</tr>
</tbody>
</table>

The fourth column shows the percent decrease in the size of the generated code that was achieved with type reduction.

Table 2
The effect of type reduction to compilation times in the five test programs

<table>
<thead>
<tr>
<th></th>
<th>Visual studio (%)</th>
<th>GCC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>106</td>
<td>116</td>
</tr>
<tr>
<td>Test 2</td>
<td>78</td>
<td>97</td>
</tr>
<tr>
<td>Test 3</td>
<td>87</td>
<td>118</td>
</tr>
<tr>
<td>Test 4</td>
<td>75</td>
<td>103</td>
</tr>
<tr>
<td>Test 5</td>
<td>194</td>
<td>307</td>
</tr>
</tbody>
</table>

The ratios are shown in percents, and computed as Tr/Tn, where Tn is the compilation time without type reduction and Tr the compilation time using type reduction.

6.4. Test results

Our results are obtained as follows: For each of the five tests in an otherwise empty program, we construct an instance of any_image with the corresponding image type set, and invoke the corresponding algorithm. We measure the size of the resulting executable, and subtract from it the size of the executable if the algorithm is not invoked (but the any_image_view instance is still constructed). The resulting difference in code sizes can thus be attributed to just the code generated from invoking the algorithm. We compute these differences for both platforms, with and without the reduction mechanism, and report the results in Table 1.

The results show that we are, on the average, cutting the executable size by more than half under GCC, and as much as 70% at times. Since Visual Studio can already avoid generating instantiations whose assembly code is identical, our gain with this compiler is less pronounced. However, we can still observe a reduction in the executable size, as much as 37% at times. We believe this is due to two factors. First, Visual Studio’s optimization cannot be applied when the code is inlined (which is the case for tests 1, 2 and 5). Indeed those tests show the largest gain. Second, the simplification of the switch statements reduces executable size—even for non-inlined code in test 3, we observed a notable reduction. Test 3 without reduction generates 11 (nested) switch statements of 10 cases each, whereas we only generate one switch statement with 26 cases. We also tried inlining resample_view under Visual Studio and got roughly 30% code reduction for tests 3 and 4, (in addition to being about 20% faster to compile, and slightly faster to execute since we avoid two function calls and a double-dispatch).

We also measured the time to compile each of the five tests of both platforms when reduction is enabled, and compared it to the time when no reduction is enabled. The results are reported in Table 2. We believe there are two main factors in play. On the one hand our reduction techniques involve some heavy-duty template meta-programming, which slows down the compiler. On the other hand, the number of instantiated copies of the algorithm is greatly reduced, which reduces the amount of work for the later phases of compiling, in particular if the algorithm’s implementation is of substantial size. In addition, a large portion of the types generated during the reduction step are not algorithm-dependent, and might be reused when another related algorithm is compiled with the same image set. Finally, when compile times are a concern, our technique may be enabled only towards the end of the product cycle.

The reduction mechanism did not result in a noticeable change in the run-time performance of our tests. The dispatch code to the algorithm changes due to the reduction mechanism, but now more indirections are performed. Also, as the dispatching occurs only once per an algorithm invocation, the run-time effect can be expected to be negligible. We do not anticipate any scenarios for which reduction could degrade run-time performance. Reduction might sometimes be beneficial for performance as smaller code size could lead to better instruction cache behavior—we did not observe this effect in our tests.
7. Conclusions

Combining run-time polymorphism and generic programming with the instantiation model of C++ is non-trivial. We show how variant types can be used for this purpose but also how, without caution, their use easily leads to a severe code bloat. As its main contribution, the paper describes library mechanisms for significantly reducing code bloat, that results from invoking generic algorithms with variant types, and demonstrates the effectiveness of the mechanisms in the context of a production quality generic library.

We discussed the problems of the traditional class-centric approach to addressing code bloat: template hoisting within class hierarchies. This approach requires third-party developers to abide by a specific hierarchy in a given module, and can be inflexible—one hierarchy may allow template hoisting for certain algorithms but not for others. Moreover, complex relationships involving two or more objects may not be representable with a single hierarchy.

We presented an alternative, algorithm-centric approach to addressing code bloat, which allows the definition of partitions among types, each specific to one or more generic algorithms. The algorithms need to be instantiated only for one representative of the equivalence class in each partition. Our technique does not enforce a particular hierarchical structure that extensions to the library must follow. The rules for type reduction are algorithm-dependent, and implemented as metafunctions. The clients of the library can define their own equivalence classes by specializing a particular type reduction template defined in a generic library, and have the induced type reductions be applied when using the generic algorithms. Also, new algorithms can be introduced by third-party developers, and all they need to do is define the reduction rules for their algorithms. Algorithm reduction rules may be reused; we discussed the copy_pixels and resample_view algorithms which have identical reduction rules.

The primary disadvantage of our technique is that it relies on a cast operation, the correctness of which is not checked. The reduction specifications declare that a given type can be cast to another given type, when used in a given algorithm. That requires intimate knowledge of the type and the algorithm. Nevertheless, we believe the generality and effectiveness of algorithm-centric type reduction justify the technique. We demonstrated that this technique can result in reducing the size of the generated code in half for compilers that do not support template bloat reduction. Even for compilers that employ aggressive pruning of duplicate identical template instantiations, our technique can result in a further noticeable decrease in code size.

The framework presented in this paper is essentially an active library, as defined by Czarnecki et al. [8]. It draws from both generic and generative programming, static metaprogramming with C++ templates in particular. We accomplish a high degree of reuse, and good performance with the generic programming approach to library design. Static metaprogramming allows us to fine tune the library’s internal implementation—for example, to decrease the amount of code to be generated.

Potential future work includes experimenting with the framework in domains other than imaging. We have experience on generic libraries for linear algebra, which seems to be a promising domain, sharing similarities with imaging: a large number of variations in many aspects of the data types (matrix shapes, element types, storage orders, etc.). A “concept analysis” in the style of the STL may prove fruitful for identifying type reductions that could be reused across library boundaries.

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